Lecture 01

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## The Josephus Problem

If people are standing in a circle and every th person is removed until only 1 person remains, at which position should we stand so that we are the last remaining person?

For example, if and , standing at position ensures that we are the last person remaining. People are removed in the order , , (since it is a circle) and finally .

## Russel’s Paradox

If a barber only shaves the people who do not shave themselves, does the barber shave themselves?

## The Liar Paradox

“This statement is a lie.”

## Probability and Stochastic Processes

**Probability** is the likelihood of an event occurring. It is a quantitative descriptor.

A **stochastic process** is a random event.

## Probabilistic and Statistical Reasoning

Reasoning can be of two types:

* **Probabilistic** – This is the classical method. It is used when we have the quantitative data.
* **Statistical** – This is through observation. We do not have any data.

## Methods of Assigning Probability

### Subjective/Judgement Method

The **subjective method**, also known as the **judgement method**, involves taking the best guess from a few events. It is the degree of belief that an outcome will occur. As such, it is an unreliable method.

### Classical Method

Using the **classical method**, the exact outcome is unknown before conducting an experiment, but all possible outcomes are known, and each outcome is **equally likely**. The experiment can be repeated to achieve the same results under uniform conditions.

For example, for a coin toss, and .

### Relative Frequency Method

The **relative frequency method** is statistical reasoning. It is used when the classical method is not possible, either because the outcomes are not equally likely or because the experiment is not repeatable under uniform conditions.

The larger the sample size, the more accurate this method is.

### Choosing a Method

We should always try to use the **classical method**. However, in some situations, such as with economic activity, the classical method cannot be used. Then, we should use the **relative frequency method**. The last choice should be the **subjective method**.

## Basic Concepts

**Probability** – The extent to which an event is likely to occur.

**Stochastic Probability** – Family of random variables.

**Experiment** – Process by which an observation or measurement is obtained, e.g., a coin toss.

**Outcomes / Events** () – Result of an experiment, e.g., heads or tails.

**Mutually Exclusive Events** – Events that cannot occur together. .

**Simple Events** – Any event consisting of a single point in the sample space.

**Sample Space** () – The set of all possible outcomes of an experiment.

**Null Event** () – Does not contain any outcomes. .

## Probability of an Event

The probability of an event is given by . If is not a simple event and consists of other events, then is the sum of all those events. The value of will always be between and . The sum of all possible simple events in the event space is .

## Counting Rules

For most sample spaces, we cannot count the possible number of events manually. Instead, we need to follow some rules to count the events.

### The Rule

If an experiment is performed in two stages and there are possibilities in the first stage and possibilities in the second stage, there are possible ways of conducting the experiment in total.

For example, if we roll a 6-sided die and then toss a coin, the total number of possible outcomes is .

### Permutations

**Permutations** are the number of different ways we can arrange objects, taking at a time.

In permutations, the order of choosing the items is relevant. For example, if there is a 3-digit lock with the numbers having to be distinct from the set , , and , there are permutations.

### Combinations

**Combinations** are like permutations in that we are taking items from a total of items, except this time the order of us picking the items is not relevant.

For example, from a deck of 52 cards if we want to pick 5 cards of which 2 have the same face value while the other 3 have different face values, the number of ways we can do this is:

* for picking 2 cards out of the 4 suites possible face values
* choices for 3 cards with different face values possible suites

Total Probability

## Event Relations

* Union -
* Intersection -
* Complement -

## Conditional Probabilities

if

Events and are **independent** if an only if and . Otherwise, they are **dependent**.

## Law of Total Probability

Let , , , be **mutually exclusive** and **exhaustive** events. In this case,

## The Multiplicative Rule for Intersections

For any events and , the probability that both events occur is

If and are mutually exclusive, .

## Bayes Rule

**Bayes Rule** states that for a set of mutually exclusive and exhaustive events, , , , , with prior probabilities , , , , if an event occurs, the posterior probability of , given that has occurred, is given by

The **prior probability** of the event is the **non-conditional probability**. The **posterior probability** of the event the **conditional probability** given that an event has occurred.

All the above equation is saying is, if we know that has occurred, the probability that has also occurred is given by the probability of occurring under divided by the sum of all possible events under which can occur, i.e., .

Consider that we know that of a population is female. Of this, are at risk of a heart attack while of males are at risk of a heart attack. Given that someone has a heart attack, what is the probability of them being male?